12-MTL-A2, TRANSPORT PHENOMENA IN MATERIALS ENGINEERING

MAY 2017

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.

3. Candidates may use any **non-communicating** calculator.

4. Regular graph papers will be provided.

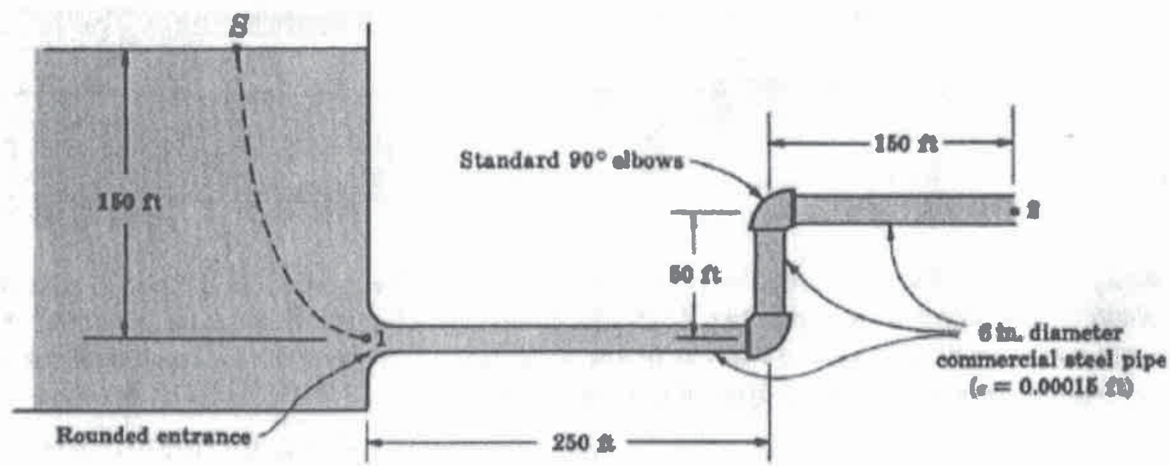
5. All problems are worth **25 points**.

6. Any **four questions** constitute a complete paper.

7. Only the **first four** questions as they appear in the answer book will be marked.

8. State all assumptions clearly.

1. Water flows from a large reservoir and discharges into the atmosphere at Point 2 as shown in the figure below:



Determine the volumetric flow rate of water discharged.

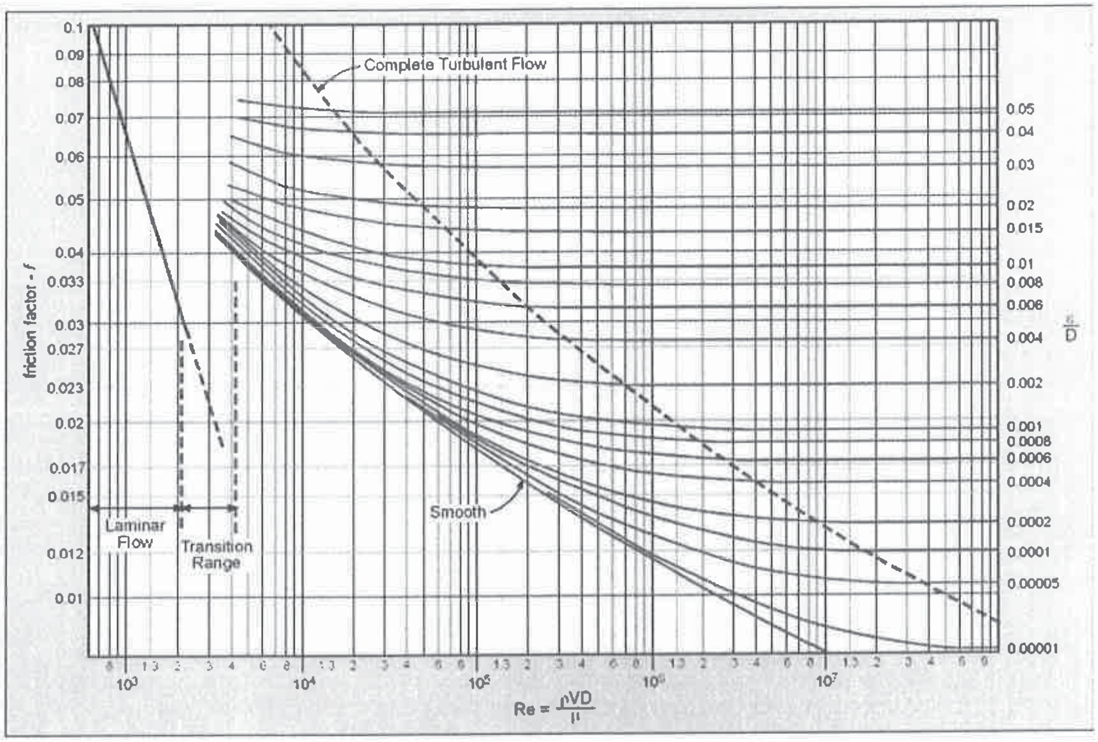
<u>DATA</u>:

Kinematic viscosity of water = 1×10^{-5} ft^2/sec

Loss coefficient for rounded entrance to a tube/pipe = 0.25

Loss coefficient for a 90° elbow = 0.90

Average roughness height (ε) of the commercial steel pipe = 0.00015 ft



Moody friction factor (f) vs. Reynolds number (Re) for pipes

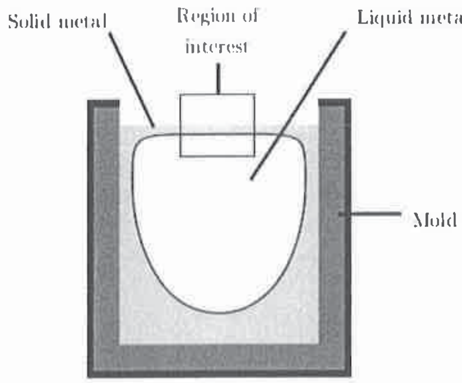
2. A sample of tracer was injected as a pulse to a reactor and effluent concentration (C) measured as a function of time shown in the following data:

Time (sec)	**Effluent Concentration of Tracer (g/cm^3)**
0	0
10	0.2
20	0.4
30	0.7
40	1.5
50	1.8
60	1.2
70	0.8
80	0.4
90	0.3
100	0.1
110	0

a) **[20 points]** Evaluate the mean residence time.

b) **[5 points]** Determine the number of stirred-tank reactors if tanks-in-series model is used to approximate the data.

3. Casting with large open tops often freezes off by radiation and convection, forming a solid shell on the top and trapping the liquid beneath it. An example is the rate of solidification downward from the top surface in a low-carbon steel ingot casting as shown below:



One can assume that the liquid metal temperature is uniform, the environment absorbs all radiation (i.e., it is black), and the top of the solid shell is grey.

a) [3 points] Estimate the thermal conductivity of iron near its melting point.

b) [2.5 points] Write an expression for the total radiative and convective heat flux from the top surface of the solidifying metal shell to the surrounding environment.

c) [3.5 points] Assuming the environment is much colder than the shell, calculate a total heat transfer coefficient, which is the ratio between heat flux and absolute temperature.

d) [4 points] Use the heat transfer coefficient from part (c) to estimate the thickness (Y) of the solid metal at which temperature can no longer be considered uniform (where the Biot number reaches a value of 0.1).

e) **[6 points]** Estimate the rate of growth of the solid while solidification rate is limited by radiation/convection from the top surface and the solid temperature can be considered uniform. Assume quasi-steady-state behavior in the solid, i.e., the heat flux is the same throughout the thickness.

f) **[6 points]** Set up equation for solidification rate limited by both radiation/convection from the top and also quasi-steady-state conduction through the solid. Do not solve the equation!!!

DATA FOR IRON:

Electrical conductivity near melting point = 5×10^5 $(\Omega.m)^{-1}$

Density = 7500 kg/m^3

Heat capacity = 500 J/kg.K

Melting point = 1800 K

Heat of fusion = 267 kJ/kg

Radiative emissivity = 0.6

Heat transfer coefficient to air = 100 $W/m^2.K$

4. In many situations, product designs include parts whose only function is to conduct or resist the flow of heat. Materials for these parts are thus chosen entirely on the basis of their thermal properties. The following candidate materials are available for consideration:

Material	**Thermal Conductivity (k),** in W/m.K	**Density (ρ),** in g/cm^3	**Heat Capacity (C_p),** in J/kg.K
Aluminum	238	2.7	917
Copper	397	8.96	386
Gold	315.5	19.3	130
Silver	425	10.5	234
Diamond	2320	3.5	519
Graphite	63	2.25	711
Lime	15.5	3.32	749
Silica	1.5	2.32	687
Alumina	39	3.96	804

Based on materials and their properties listed above, select the best material for each of the following applications and briefly explain your selection:

a) **[4 points]** Heat shield sandwiched between a hot body and a cold one which minimizes the steady flux between them.

b) **[4 points]** Heat shield that protects something from short, intense bursts of heat (long timescale is needed).

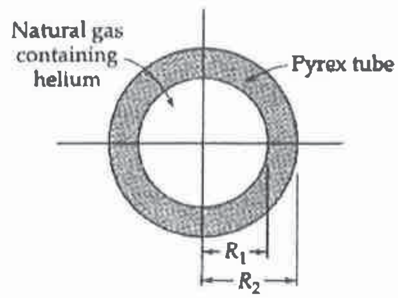
c) **[3 points]** Cheap temperature sensor (do not list diamond!!!) in which short timescale of heat conduction is necessary for rapid response.

d) **[4 points]** Light heat reservoir, which must hold as much heat as possible per degree C per unit weight.

e) **[4 points]** Heat sink for a semiconductor device which must minimize temperature difference for a given heat flux.

f) **[6 points]** Economically viable heat sink for melt spinning in which liquid metal is injected against a rotating heat sink where it is solidified as rapidly as possible so the material must conduct heat away from the surface quickly.

5. Pyrex glass is almost impermeable to all gases except helium. A method for separating helium from natural gas could be based on the relative diffusion rates through Pyrex. Suppose a natural gas mixture is contained in a Pyrex tube as shown in the figure below:



Obtain an expression for the rate at which helium will "leak" out of the tube in terms of diffusivity of helium through Pyrex, interfacial concentrations of helium in the Pyrex, and dimensions of the tube.

APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho\vec{u}) = 0$$	(1.1)
Rectangular coordinates (x, y, z) $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$$	(1.1a)
Cylindrical coordinates (r, θ, z) $$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0$$	(1.1b)
Spherical coordinates (r, θ, ϕ) $$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi) = 0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla P + \vec{g} + \nu(\nabla^2 \vec{u})$$	(A2)
Rectangular coordinates (x, y, z)		
x-component	$$\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g_x + \nu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + g_y + \nu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z + \nu\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \quad \text{(A2d)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \quad \text{(A2e)}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \quad \text{(A2f)}$$

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r +\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2 u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \quad \text{(A2g)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta +\nu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\right] \quad \text{(A2h)}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta = -\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi} +g_\varphi+\nu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\right] \quad \text{(A2i)}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=[\nabla\cdot k\nabla T]+\dot{T}_G$$	(A3)
Rectangular coordinates (*x*, *y*, *z*) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$$	(A3a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$$	(A3b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=$$ $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G$$	(A3c)

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-(\nabla\cdot\vec{N}_A)+\dot{R}_{A,G}$$	(4.)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_x}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G}$$	(4a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G}$$	(4b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G}$$	(4c)

Table A.5: The continuity equation for species *A*

$$\frac{\partial C_A}{\partial t}+(\vec{u}\cdot\nabla)C_A=D_A\nabla^2C_A+\dot{R}_{A,G} \qquad (5)$$
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t}+u_x\frac{\partial C_A}{\partial x}+u_y\frac{\partial C_A}{\partial y}+u_z\frac{\partial C_A}{\partial z}=\frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right)+\frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right)+\frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right)+\dot{R}_{A,G} \qquad (5a)$$
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t}+u_r\frac{\partial C_A}{\partial r}+\frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta}+u_z\frac{\partial C_A}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right)+\dot{R}_{A,G} \qquad (5b)$$
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t}+u_r\frac{\partial C_A}{\partial r}+\frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2D\frac{\partial C_A}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right)+\dot{R}_{A,G} \qquad (5c)$$