National Exams December 2016

98-Ind-A1

Operations Research

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used; i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.

3. All questions are worth 20 marks; therefore, the total value of all 8 questions is 160 marks. Any marks achieved will be considered toward the 100 total requirement.

20 1. Joe is selling Christmas trees to pay for his college tuition. He purchases trees for $10 each and sells them for $25 each. The number of trees he can sell is normally distributed with a mean of 100 and a standard deviation of 30. How many trees should Joe purchase?

20 2. The sales manager of a publisher of university textbooks has six travelling sales staff to assign to three different regions. She has decided that each region should be assigned one or more dedicated sales staff. The estimated sales per region varies with the number of staff assigned as follows:

No. of sales staff	Region 1	Region 2	Region 3
1	35	21	28
2	48	42	41
3	60	56	53
4	69	70	65

Use dynamic programming to determine how the six travelling sales staff should be assigned to the three regions to maximize the total sales.

20 3. An oil company is installing an oil pipeline from an oil field to a refinery. The pipeline requires the welding of 1000 seams, to be carried out by the company's own welders. Defective seams result in leaks, which must be reworked at a cost of $1,200 per seam. It is estimated from past experience that 5% of the seams will be defective with probability 0.30, or 10% will be defective with probability 0.50, or 20% will be defective with probability 0.20. The company can also hire an expert cleanup team of welders at a one-time cost of $130,000, who would check all of the welds done by the company welders and repair them as required.

a. Based on an expected value criterion, should the company bring in the expert clean-up team to check and rework the welds, or repair the welds as they occur?

b. The company can also improve its information about the quality of its own welders on this job, by x-ray inspection of a randomly selected completed weld at a cost of $2,000. Is it worthwhile to carry out this inspection?

20 4. Alexis Cornby makes her living buying and selling corn. On January 1st, she has 50 tons of corn and $1,000. On the first day of each month Alexis can buy corn at the following prices per ton: January, $300; February, $350; March, $400; April, $500. On the last day of each month, Alexis can sell corn at the following prices per ton: January, $250; February, $400; March, $350; April, $550. Alexis stores her corn in a warehouse that can hold at most 100 tons of corn. She must be able to pay cash for all corn at the time of purchase. Formulate the linear program which will determine how Alexis can maximize her cash on hand at the end of April.

5. Consider the following problem

Maximize $z = 21x1 + 9x2 + 4x3$ (profit)

Subject to

$2x1 + x2 + x3 \leq 31$ (resource constraint 1)
$3x1 + 2x2 + x3 \leq 60$ (resource constraint 2)
$x1 + 2x2 + x3 \geq 50$ (requirement constraint)
$x1 \geq 0$
$x2 \geq 0$

The simplex method yields the following final set of equations

$z + (1/2)x3 + (2/3)x4 + x6 = 291$
$x1 + (1/3)x3 + (2/3)x4 + (1/3)x6 = 4$
$x2 + (1/3)x3 - (1/3)x4 - (2/3)x6 = 23$
$x5 - (2/3)x3 - (4/3)x4 + (1/3)x6 = 2$

where x4 is the slack variable for resource constraint 1, x5 is the slack variable for resource constraint 2, and x6 is the slack variable for the requirement constraint.

6 a. What is the optimal solution, the maximum profit, the marginal values of resources 1 and 2, and the marginal cost of the requirement?

7 b. How much can the coefficient of x2 in the objective function vary without affecting the optimal solution?

7 c. By how much would the profit be increased if 5 more units of resource 1 where available? What would be the new solution?

20 6. Consider an electricity company with a 100 MW generating unit. The unit has a long history of failures which often restrict its output to 75, 50, 25 or 0 MW. After examining the historical records, you determine that the probability of failures is only a function of the current state, and does not depend in any way on what happened before. Thus you believe a Markov Chain model is appropriate. Whenever it is producing any output there is a 5% probability that in the next hour it will totally fail and be forced out of service. After one hour of repair there is a 50% chance that it will still be unable to produce any electricity in the following hour, a 40% chance it will return to produce 100 MW in the next hour, a 6% chance it will produce 75 MW, a 3% chance it will produce 50 MW and a 1% chance it will produce only 25 MW. What is the long-term expected output of this unit?

7. Before a new product can be introduced the following activities must be completed (all times are in weeks):

Activity	Predecessor	Duration	Speed-up cost in $/wk
A	-	6	80
B	-	5	60
C	A	3	30
D	C	2	60
E	A,D	3	40
F	B	2	30
G	E	4	20
H	G, F	2	-

5 a. total float and free float for each activity

5 b. Set up an LP that can be used to determine the critical path

5 c. Formulate a minimum cost network flow problem that can be used to find the critical path

5 d. It is now 12 weeks before Christmas. The duration of each activity can be reduced by up to two weeks by incurring the Speed-up costs shown in the table. Formulate a Linear Programming model that will minimize the cost of getting the product into the stores before Christmas.

20 8 An electric utility is considering 5 possible locations to build additional power plants over the next 20 years. The cost of building a plant at each site, the annual operating cost and the energy/year that can be provided at each site are given, as well as the total energy requirement for each year of the planning horizon. Assume that at most one plant can be placed into service in a given year, and that it can produce its full energy contribution starting the year it is placed in service. The company can currently generate 500,000 kwh/year, using its existing resources. Develop an integer programming model to determine the expansion plan that will minimize overall construction and operating costs over the 20 years.