National Exams May 2014

07-Mec-B6, Advanced Fluid Mechanics

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any non-communicating calculator. The exam is OPEN BOOK.
3. PART A: Answer any **3** of the 4 questions. PART B: Answer and **2** of the 3 questions.
4. Weighting: PART A: 42%; PART B: 58%. Within each section, questions are equally weighted.

PART A: Answer any **3** of the following 4 questions.

Question A1: A rectangular, thin metal flat plate has dimensions a = 5m and b = 1m. The plate is to be pulled in a still fluid at a constant rate of U_o = 15 m/s. The plate can be pulled either with side a or side b normal to the flow (arrangements **A** and **B**, respectively, in the Fig. A1). Determine for each arrangement the drag (force needed to pull the plate) and the power required. Identify which arrangement is more energy efficient and then estimate the percentage reduction in power requirement. Assuming that laminar-turbulent transition occurs at a critical Reynolds number of 500,000, conduct the analysis for:

a) Water (ρ = 1000 kg/m^3; ν = 10^{-6} m^2/s) and
b) Air (ρ = 1.2 kg/m^3; ν = 15 × 10^{-6} m^2/s).

Neglect gravitational effects.

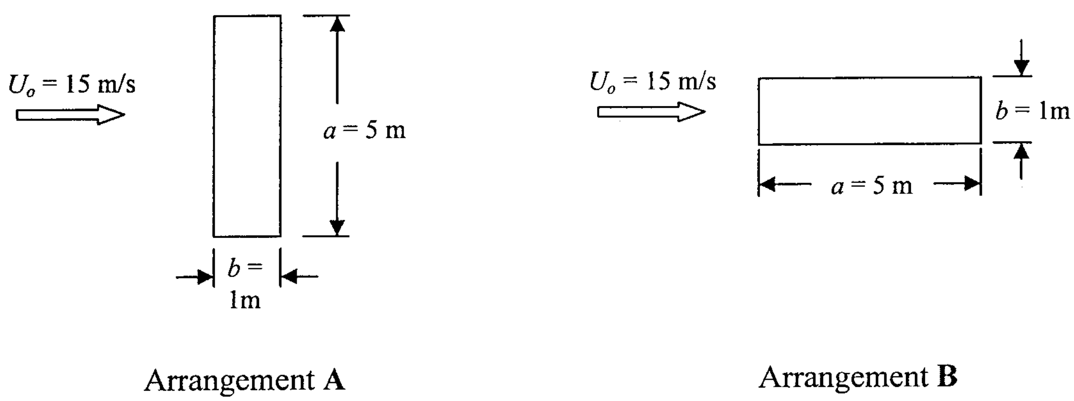


Figure A1: Two arrangements for a plate being pulled at a constant rate U_o in a still medium.

Question A2: Aircraft commonly use pitot-static tubes to determine their airspeed. If an aircraft is flying at 11,000m (pressure of still air is 30kPa, temperature is -55°C) and the pitot-static tube readings yield a stagnation pressure of 51kPa, determine the airspeed of the aircraft. What would be the error in the speed estimated if the Bernoulli equation were blindly used? For air, use R = 287 J/kg-K, $\gamma = 1.4$.

Question A3: Hydrogen is pumped through a 2m inner diameter pipeline from Edmonton to Calgary (300km). The gas leaves the first booster station at a mass flow rate of 74.3 kg/s and static conditions of -30°C and 1MPa. Determine the maximum distance between booster stations. Given that each station has a compressor and cooler to maintain the same outlet conditions as the first booster station and that the total-to-total efficiency of the compressor is 0.85, determine the power consumption for each compressor and the heat rejected by each cooler. At what static pressure and temperature is the hydrogen delivered?

The friction factor for the pipeline is given as 0.03 and the system may be considered well insulated. For hydrogen use: $\gamma = 1.4$, R = 4.124 kJ/kg-K, Cp = 14.35 kJ/kg-K.

Question A4: You are working for a consulting engineering firm which has been contracted to predict the loading on a simple suspension bridge. The bridge has a deck length of 50m and a deck width of 5m. To test the bridge, you construct a model at 20:1 scale to be tested in a water tunnel.

a) If the loading is to be tested to match an equivalent wind (air) speed of 30m/s, what must be the speed of the water at which the model is tested?
b) If the lift and drag forces in the water model are determined to be 1.0 MN and 0.1 MN, respectively, what will be the predicted loading on the full scale bridge?
c) If, in the water, the bridge oscillates at a frequency of 5 Hz, estimate the oscillation frequency in air.

Solve the problem using full-similarity. For air, use: $\rho = 1.2$ kg/m^3 ; $\mu = 18 \times 10^{-6}$ Pa-s. For water, use: $\rho = 1000$ kg/m^3 ; $\mu = 0.001$ Pa-s.

PART B: Answer any <u>2</u> of the following 3 questions.

Question B1: A large reservoir contains air (R = 287 J/kg-K; $\gamma = 1.4$) at the total pressure P_o and total temperature T_o. It is connected to a pipe through a convergent-divergent nozzle. The pipe has constant cross-sectional area A_1 = 10.0 cm^2. At the exit of this nozzle, it is determined that the flow is supersonic and the mean flow rate is 0.175 kg/s. The air exits through a second convergent-divergent section with a throat with a cross-sectional area of A_T = 7.75 cm^2 and an exit area of A_3 = 15.19 cm^2. At the exit, the ambient (back) pressure is P_3 = 100 kPa and the Mach number is $M_3 = 0.3$. Assuming that frictional losses are negligible and the system is well-insulated, determine:

a) The air temperature and total pressure at the exit.
b) The total pressure, P_o. and total temperature, T_o, in the reservoir.
c) The static conditions (P_T, T_T, M_T) in the throat.
d) The minimum throat area that would allow the flow conditions to exist.

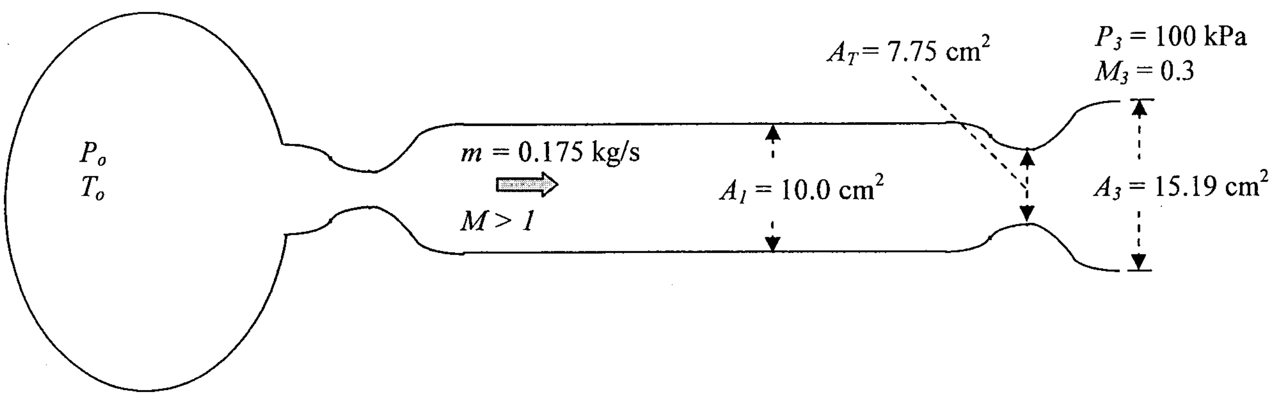


Figure B1: Schematic of high-speed test section.

Question B2: A very long, vertical pipe of inner diameter D is filled with an incompressible, Newtonian fluid (density ρ and dynamic viscosity μ). A solid cylindrical rod of diameter d is placed at the centre and pulled upwards at a constant rate U_p. The pressure gradient in the axial (vertical) direction is zero and the flow may be assumed fully developed and laminar. The fluid wets the walls of the pipe and rod. The walls are non-porous.

a) State the boundary conditions for the velocity components.
b) Find the expression for the radial velocity component, u_r.
c) Find the expression for the radial distribution of the axial velocity component, u_z.
d) What is the force needed (per unit length) to pull the rod? What is the force (per unit length) on the pipe?

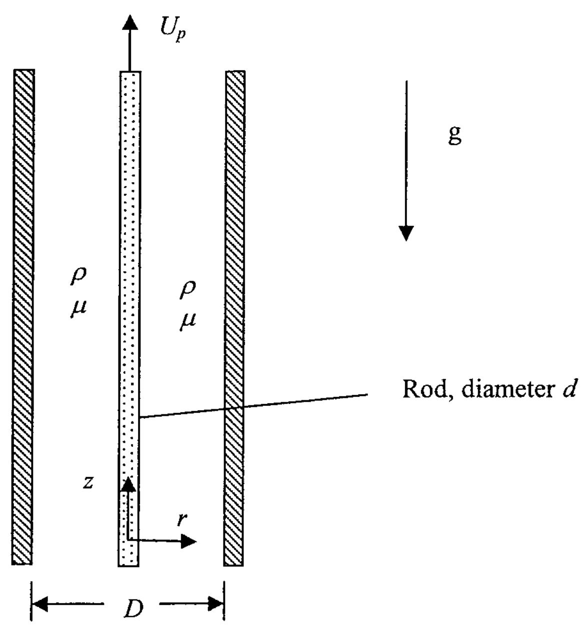


Figure B2: Schematic of vertical pipe with rod.

Question B3: A two dimensional sink of strength m and a source of strength $2.5m$ are placed vertically at a distance b and $2b$, respectively from a solid wall. Assuming that the fluid (density ρ) is ideal and the flow is irrotational, determine:

a) A suitable potential or stream function to model this flow. Explicitly show that the wall is correctly represented.
b) The velocity distribution and the stagnation points along the wall. Sketch the u-velocity along the wall.
c) The pressure distribution along the wall (use P_o to denote the pressure very far away from the wall) and indicate the points of maximum pressure.

Neglect gravitational effects.

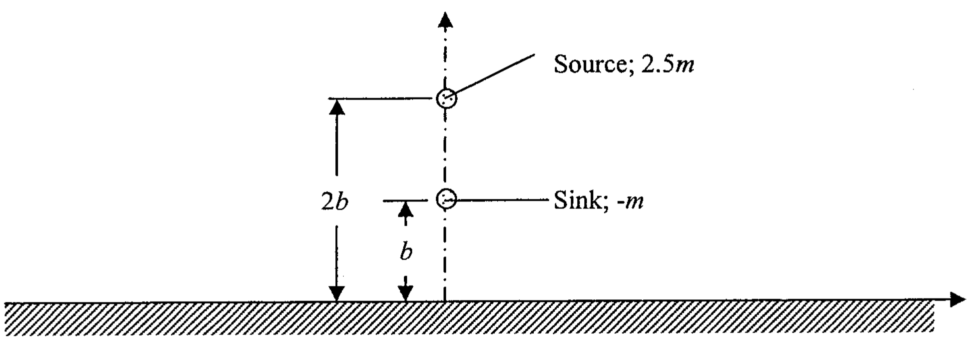


Figure B3: Schematic of sink and source in vertical tandem arrangement.

Aid Sheets

Compressible Flow:

Adiabatic flow: $$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow: $$\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \quad ; \quad \frac{\rho_o}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

$$\dot{m} = \rho U A = \sqrt{\frac{\gamma}{R}} \cdot \frac{P_o}{\sqrt{T_o}} \cdot M \cdot \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma + 1}{2(\gamma - 1)}} A$$

Shock Relations: $$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^{\ 2}}{1 + \gamma M_2^{\ 2}} = \frac{2\gamma M_1^{\ 2}}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} \qquad M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

Boundary Layer Equations:

$$\frac{d}{dx}\left(\mathrm{U}_o^2 \theta\right) + \mathrm{U}_o \frac{d\mathrm{U}_o}{dx} \cdot \delta^* = \tau_w / \rho \qquad \delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy \qquad \theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Laminar flow: $$C_{fx} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2} = \frac{0.67}{\mathrm{Re}_x^{1/2}}$$

Turbulent flow: $$C_{fx} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2} = \frac{0.0266}{\mathrm{Re}_x^{1/7}}$$

Conservation Equations for Cartesian Coordinate system

Continuity Equation:

$$\frac{D\rho}{Dt} + \rho\left(\nabla \bullet \vec{U}\right) = 0$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} \qquad \text{and} \qquad \nabla \bullet \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Linear Momentum:

***x*-direction:** $$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial P}{\partial x} + \rho g_x + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$$

***y*-direction:** $$\rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} + \rho w\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + \rho g_y + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}$$

***z*-direction:** $$\rho\frac{\partial w}{\partial t} + \rho u\frac{\partial w}{\partial x} + \rho v\frac{\partial w}{\partial y} + \rho w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + \rho g_z + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu\nabla \bullet \vec{U} \qquad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\tau_{yy} = 2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu\nabla \bullet \vec{U} \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \qquad \tau_{zz} = 2\mu\frac{\partial w}{\partial z} - \frac{2}{3}\mu\nabla \bullet \vec{U}$$

Conservation Equations for Cylindrical-polar Co-ordinate system

Continuity Equation: $$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta)+\frac{\partial}{\partial z}(\rho u_z)=0$$

Linear Momentum Equations:

***r*-momentum:**

$$\rho\left[\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}\right]$$
$$=-\frac{\partial P}{\partial r}+\rho g_r+\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})+\frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{r\theta})+\frac{\partial}{\partial z}(\tau_{rz})-\frac{\tau_{\theta\theta}}{r}$$

θ-momentum:

$$\rho\left[\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}\right]$$
$$=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\rho g_\theta+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{\theta r})+\frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{\theta\theta})+\frac{\partial}{\partial z}(\tau_{\theta z})$$

***z*-momentum:**

$$\rho\left[\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right]$$
$$=-\frac{\partial P}{\partial z}+\rho g_z+\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{zr})+\frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{z\theta})+\frac{\partial}{\partial z}(\tau_{zz})$$

$$\tau_{rr}=\mu\left(2\frac{\partial u_r}{\partial r}-\frac{2}{3}\nabla\cdot\vec{U}\right) \qquad \tau_{\theta\theta}=\mu\left(2\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r}{r}\right)-\frac{2}{3}\nabla\cdot\vec{U}\right)$$

$$\tau_{zz}=\mu\left(2\frac{\partial u_z}{\partial z}-\frac{2}{3}\nabla\cdot\vec{U}\right) \qquad \tau_{r\theta}=\tau_{\theta r}=\mu\left(r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)+\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$

$$\tau_{rz}=\tau_{zr}=\mu\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right) \qquad \tau_{\theta z}=\tau_{z\theta}=\mu\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta}+\frac{\partial u_\theta}{\partial z}\right)$$

$$\nabla\cdot\vec{U}=\frac{1}{r}\frac{\partial}{\partial r}(r u_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta)+\frac{\partial}{\partial z}(u_z)$$

$$\nabla\times\vec{U}=\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta}-\frac{\partial u_\theta}{\partial z}\right)\cdot\vec{e_r}+\left(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right)\cdot\vec{e_\theta}+\left(\frac{1}{r}\frac{\partial(r u_\theta)}{\partial r}-\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)\cdot\vec{e_z}$$

Potential Flow

Stream functions:

Uniform Flow: $$\Psi = U_o y = U_o r \sin\theta$$

Source Flow: $$\Psi = \frac{m}{2\pi}\tan^{-1}\left(\frac{y-y_o}{x-x_o}\right) = \frac{m}{2\pi}\theta$$

Vortex Flow: $$\Psi = -\frac{\Gamma}{4\pi}\ln\left[(x-x_o)^2+(y-y_o)^2\right] = -\frac{\Gamma}{2\pi}\ln r$$

Doublet Flow: $$\Psi = -\frac{\lambda(y-y_o)}{(x-x_o)^2+(y-y_o)^2} = -\lambda\frac{\sin(\theta)}{r}$$

Potential functions:

Uniform Flow: $$\Phi = U_o x = U_o r \cos\theta$$

Source Flow: $$\Phi = \frac{m}{4\pi}\ln\left[(x-x_o)^2+(y-y_o)^2\right] = \frac{m}{2\pi}\ln r$$

Vortex Flow: $$\Phi = \frac{\Gamma}{2\pi}\tan^{-1}\left(\frac{y-y_o}{x-x_o}\right) = \frac{\Gamma}{2\pi}\theta$$

Doublet Flow: $$\Phi = \frac{\lambda(x-x_o)}{(x-x_o)^2+(y-y_o)^2} = \lambda\frac{\cos(\theta)}{r}$$

Velocity relationships:

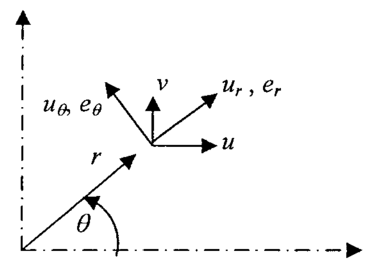
$$u = \frac{\partial\Phi}{\partial x} = \frac{\partial\Psi}{\partial y} \qquad v = \frac{\partial\Phi}{\partial y} = -\frac{\partial\Psi}{\partial x}$$

$$u_r = \frac{\partial\Phi}{\partial r} = \frac{1}{r}\frac{\partial\Psi}{\partial\theta} \qquad u_\theta = \frac{1}{r}\frac{\partial\Phi}{\partial\theta} = -\frac{\partial\Psi}{\partial r}$$

Transformation between Coordinate System

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = r\cos\theta \qquad y = r\sin\theta$$



$$\vec{e}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j} \qquad \vec{e}_\theta = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j}$$

$$u_r = u\cos\theta + v\sin\theta \qquad u_\theta = -u\sin\theta + v\cos\theta$$

$$u = u_r\cos\theta - u_\theta\sin\theta \qquad v = u_r\sin\theta + u_\theta\cos\theta$$