National Exams Dec 2018

16-Mec-B12, Robotics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.
2. This is a CLOSED BOOK EXAM with one 8.5"x11" formula sheet allowed written on both sides. The formula sheet must not hold any solutions of examples and must be handed in with the exam submission. An approved Casio or Sharp calculator is permitted.
3. **FIVE (5)** questions constitute a complete exam paper. The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value.
5. Question value in marks is shown in parentheses at the end of each question part.
6. Logical order, clarity, and organization of the solution steps are important.

Nomenclature:

The unit vectors of coordinate system *A* are denoted by $\hat{X}_A, \hat{Y}_A, \hat{Z}_A$.

The leading superscript such as *A* in ${}^{A}V$ indicates the coordinate system to which the vector *V* is referenced.

The leading subscript and superscript such as *A* and *B* in ${}^{B}_{A}T$ indicate the transformation of coordinate frame *A* relative to *B* by matrix *T*.

(20)

1. Explain what the columns of a rotation matrix, ${}_{B}^{A}R$ represent. Is the following matrix is a valid rotation matrix? Explain necessary test(s) for finding the answer.

$$ {}_{B}^{A}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 \\ 0 & -1/2 & \sqrt{3}/2 \end{bmatrix} $$

(20)

2. A position vector is given by ${}^{A}P = [10 \quad -4 \quad 3]^{T}$ and a velocity vector by ${}^{B}V = [15 \quad 5 \quad 20]^{T}$. Given the Homogeneous Transformation

$$ {}_{B}^{A}T = \begin{bmatrix} 0.5 & 0 & -\sqrt{3}/2 & 5 \\ -\sqrt{3}/2 & 0 & -0.5 & -10\sqrt{3}/2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}; $$

Compute:

a) ${}^{B}P$ **(12)**

b) ${}^{A}V$ **(8)**

(20)

3. The following frame definitions are given

$$ {}_{A}^{U}T = \begin{bmatrix} 0.866 & -0.5 & 0 & 11 \\ 0.5 & 0.866 & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}_{A}^{B}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.866 & -0.5 & 10 \\ 0 & 0.5 & 0.866 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}_{U}^{C}T = \begin{bmatrix} 0.866 & -0.5 & 0 & -3 \\ 0.433 & 0.75 & -0.5 & -3 \\ 0.25 & 0.433 & 0.866 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

a) Draw a frame diagram to show their arrangement graphically. **(5)**

b) Solve for ${}^{B}_{C}T$. **(15)**

(20)

4. Consider the arm with three degrees of freedom shown in Figure 1. Joint 2's axis is offset from joint 1's axis by the link length l1.

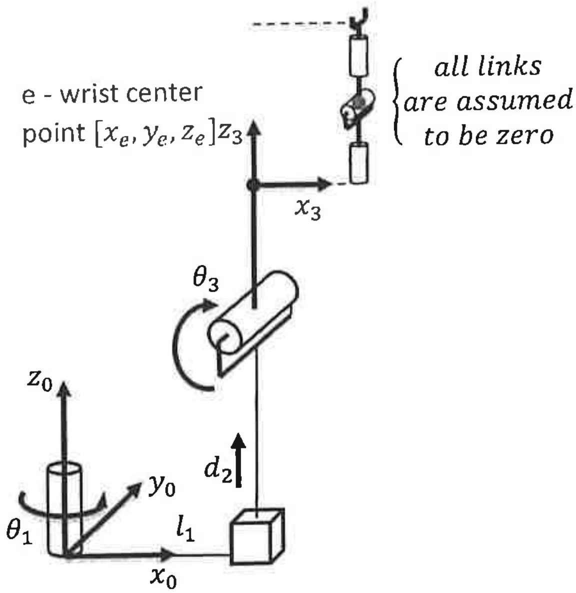

Figure 1: RPR robot

a) Affix frames in a sketch **(5)**
b) Derive link parameters (DH-Table) **(5)**
c) Derive kinematic equations for ${}^{B}_{W}T$ (the base and wrist frames). Note that no l_3 need to be defined. **(10)**

(20)

5. Two 3R mechanisms are pictured in Figure 2. In both cases, the three axis intersect at a point (and, over all configurations, this point remains fixed in space). The mechanism shown in Figure 2(a) has link twists (α_i) of magnitude 90°. The mechanism in Figure 2(b) has one twist of ϕ in magnitude and the other of $180^o - \phi$ in magnitude.

The mechanism in Figure 2(a) can be seen in correspondence with Z-Y-Z Euler angles, and therefore we know that it suffices to orient link 3 (with arrow in figure) arbitrarily with respect to the fixed link 0. Because ϕ is not equal to 90°, it turns out that the other mechanism cannot orient link 3 arbitrarily.

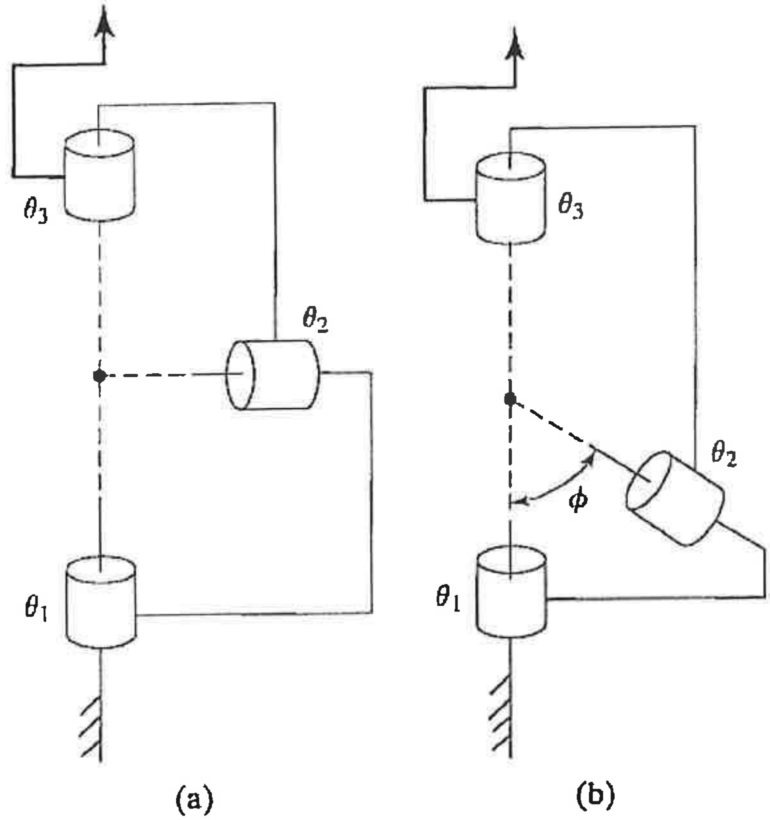

Figure 2

Describe a set of orientations that are unattainable with the second mechanism using geometric approach.

Note: we assume that all joints can turn 360° (i.e. no limits) and we assume that the links may pass through each other if need be (i.e. workspace not limited by self-collisions).

(20)

6. A certain two-link planar manipulator has the following Jacobian:

$$^{0}J(\Theta)=\begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}.$$

a) Ignoring gravity, what are the joint torques required so that the manipulator will apply a static force vector $^{0}F = 10\hat{X}_o$ where $\hat{X}_0 = [1 \quad 0]^T$? **(10)**

b) Given the above Jacobian, investigate the singular configuration(s). **(10)**

(20)

7. Solve for the coefficients of two cubics that are connected in a two-segment spline with continuous acceleration at the intermediate via point. The initial angle is $\theta_0 = -20°$, the via point is $\theta_v = 45°$, and the goal point is $\theta_g = 25°$. The first cubic is:

$$\theta(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$$

and the second is:

$$\theta(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$$

Each cubic will be evaluated over an interval starting at $t_{i1} = 0, t_{i2} = 0$ and ending at $t_{f1} = 4s$ and $t_{f2} = 4s$.