Professional Engineers Ontario

National Exams- May, 2013

Applications of the Finite Element Method

3 hours duration

Notes:

1. There are 4 pages in this examination.

2. If doubt exists as to the interpretation of any question, the candidate is urged *to submit* with the answer paper, a clear statement of any assumptions made.

3. This is a closed book examination.

4. Candidates may use one of two calculators, any Casio or Sharp approved model.

5. **Answer only three (3) problems out of the four (4) proposed.** The first three problems as they appear in the answer book will be marked.

6. All problems are of equal value.

7. One aid sheet 8.5 " x 11" written on both sides is permitted.

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Problem 1

A copper rod 1.4 inches in diameter is placed in an aluminum sleeve with inside diameter 1.42 inches and wall thickness of 0.2 inches as shown in Figure 1. The rod is 0.005 inches longer than the sleeve. A load P = 60, 000 lb is applied to the assembly through a large bearing plate that can be considered rigid. Determine the stresses in the rod and the sleeve. The modulus of elasticity for copper is 17×10^{6} psi and that for aluminum is 10×10^{6} psi.

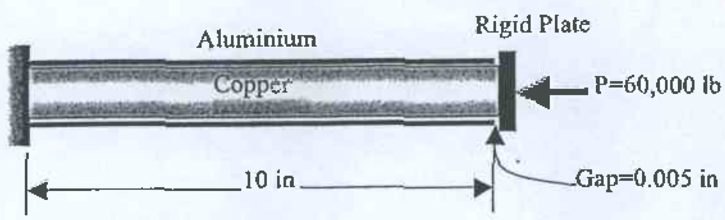


Figure 1

Problem 2

Referring to the frame shown in Figure 2(a); all members are assumed not extensible (in the axial direction) with bending rigidity EI.

2.1 Using a plane frame element stiffness formulation (given below),
(a) find the displacement and the rotations at all nodal points
(b) draw the shear force and bending moment diagrams

2.2 The frame is now reinforced with a truss bars as shown in Fig. 2(b), Draw the deflections shape, shearing forces diagram and bending moment diagram. The stiffness of the truss member is given by: $EA = \frac{12EI}{L^2}$. Comment the effect of the reinforcement.

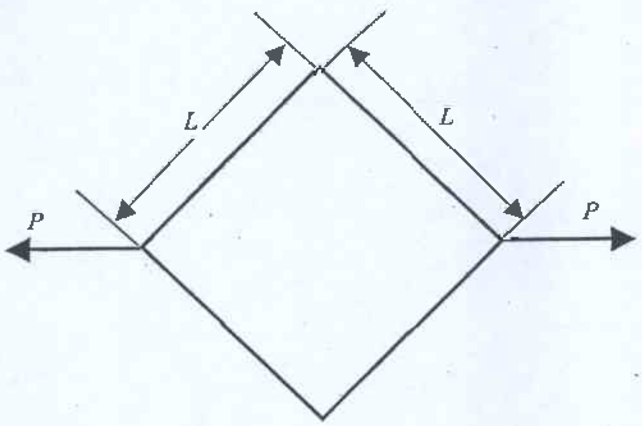


Figure 2(a)

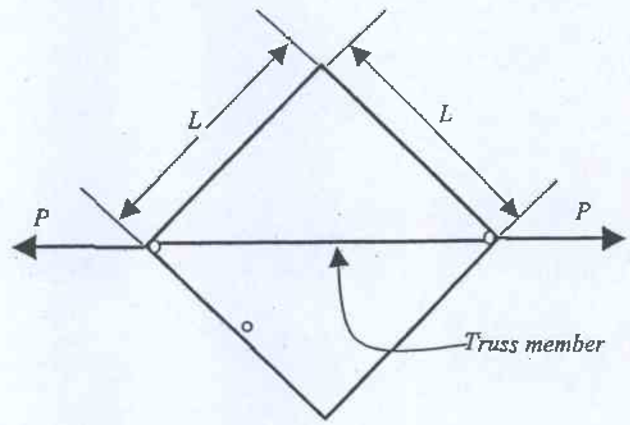


Figure 2(b)

The stiffness matrix of a beam element is given as following:

$$[k] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & \text{SYM} & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix}$$

Problem 3

For the plane strain element shown in Figure 3, determine the element stresses $\sigma_x, \sigma_y, \tau_{xy}$ and the principal stresses, corresponding to the following nodal displacements:

$$u_1 = 0.005\,mm \quad v_1 = 0.002\ mm$$
$$u_2 = 0.0\,mm \quad v_2 = 0.0\ mm$$
$$u_3 = 0.005\,mm \quad v_3 = 0.0\ mm$$

Use $E = 70\,GPa$ and $\nu = 0.3$. The coordinates of the nodes are given in Table 1.

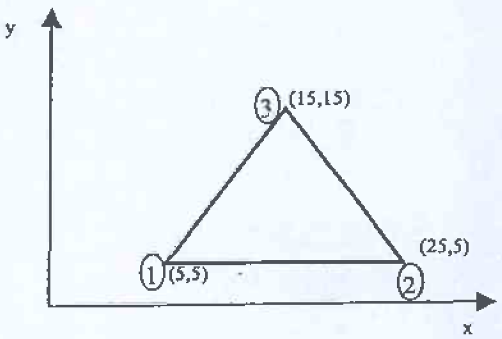


Figure 3 (all dimensions are in millimetres)

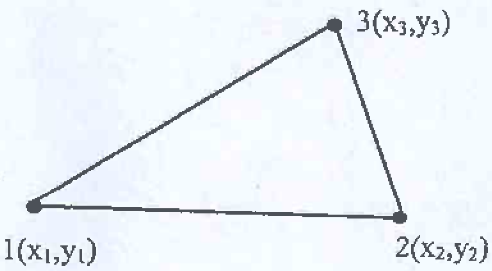
Table 1: Coordinates of the nodes

node	x	y
1	5	5
2	25	5
3	15	15

Given

The gradient matrix, [B], of a Constant Stress Triangle element (6 dof)

$$[B] = \frac{1}{2A}\begin{bmatrix} \beta_{23} & 0 & \beta_{31} & 0 & \beta_{12} & 0 \\ 0 & \alpha_{32} & 0 & \alpha_{13} & 0 & \alpha_{21} \\ \alpha_{32} & \beta_{23} & \alpha_{13} & \beta_{31} & \alpha_{21} & \beta_{12} \end{bmatrix}$$



$\beta_{ij} = y_i - y_j$ and $\alpha_{ij} = x_i - x_j$

(May, 2013)

Problems 4:

The four-node element shown was initially a square, two units on the side. Nodal displacement components, each of magnitude c in both x and y directions, create the displacement shape indicated by dashed lines (see table 2).

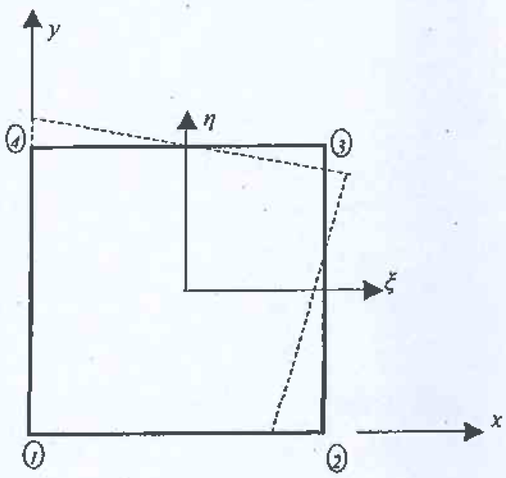


Table 2: Nodal displacements

node	x	y
1	0	0
2	-c	0
3	c	-c
4	0	c

4.1 Determine the strains at the center of the element.
4.2 Discuss these results

Given:

The shape functions on the element of reference are given by

$$\begin{cases} N_1 = \frac{1}{4}(1-\xi)(1-\eta) & N_2 = \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 = \frac{1}{4}(1+\xi)(1+\eta) & N_4 = \frac{1}{4}(1-\xi)(1+\eta) \end{cases}$$