National December 2018

16-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four (4) questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Question 1:

Consider the second-order system with the transfer function,

$$G(s) = \frac{3}{(s^2 + 2s - 3)}$$

a) Determine the DC gain for this system.

b) What is the final value of the step response of this system?

Question 2:

A typical transfer function for a tape-drive system would be (with time in milliseconds),

$$G(s) = \frac{k(s+4)}{s[(s+0.5)(s+1)(s^2+0.4s+4)]}$$

From Routh's criterion, what is the range of *k* for which this system is stable if the characteristic equation is 1+G(*s*) = 0 ?

Question 3:

A unity feedback system has the plant transfer function

$$kG(s) = \frac{k}{s(s^2 + 6s + 12)}$$

We wish to investigate the root locus versus *k*.

a) Plot the location of poles and zeros of G(s) showing segments of the root locus on the real axis,
b) What are the departure angles from the complex poles?
c) Where are the breakaway and break-in points?
d) Sketch the root locus,
e) What is the value of k at the point where the closed -loop complex roots have damping ratio ζ =0.5?

Question 4:

For the unity feedback system with

$$G(s)=\frac{1}{s(s+1)\left[(s^2/25)+0.4(s/5)+1\right]}$$

a) Draw the Bode plots (gain and phase) for G($j\omega$).
b) Indicate the gain margin when the gain is set for a phase margin of 45°.

Question 5:

a) Using the Laplace transform technique, find the transient and steady-state responses of the system described by the differential equation

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=1$$

With initial conditions

$$y(0^+) and \frac{dy}{dt}\Big|_{t=0^+}=1$$

b) Using the LaPlace transform technique, find the unit impulse response of the system described by the differential equation

$$\frac{d^3y}{dt^3}+\frac{dy}{dt}=x$$

Question 6:

a) What is the step response of a system whose transfer function has a zero at -1, a pole at -2, and a gain factor of 2?

b) Determine the time response y(t) for the following transformed equation:

$$Y(S)=\frac{s+4}{s(s+1)(s+2)}$$

Question 7:

Determine the position, velocity and acceleration error constants and then determine the steady-state error to a unit step, a unit ramp, and a unit parabolic input for the system shown below.

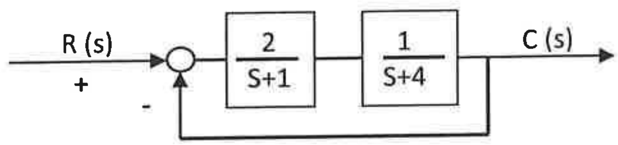

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
1	$\frac{1}{s}$	(1)
$e^{at}f(t)$	$F(s-a)$	(2)
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$	(3)
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$	(4)
$\delta(t)$	1	(5)
$\delta(t-t_0)$	e^{-st_0}	(6)
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$	(7)
$f'(t)$	$sF(s) - f(0)$	(8)
$f^n(t)$	$s^n F(s) - s^{(n-1)}f(0) - \cdots - f^{(n-1)}(0)$	(9)
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	(10)
$t^n \ (n = 0, 1, 2, \dots)$	$\frac{n!}{s^{n+1}}$	(11)
$t^x \ (x \geq -1 \in \mathbb{R})$	$\frac{\Gamma(x+1)}{s^{x+1}}$	(12)
$\sin kt$	$\frac{k}{s^2+k^2}$	(13)
$\cos kt$	$\frac{s}{s^2+k^2}$	(14)
e^{at}	$\frac{1}{s-a}$	(15)
$\sinh kt$	$\frac{k}{s^2-k^2}$	(16)
$\cosh kt$	$\frac{s}{s^2-k^2}$	(17)
$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$	(18)
$\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$	(19)
te^{at}	$\frac{1}{(s-a)^2}$	(20)
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(21)
$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$	(22)
$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$	(23)
$e^{at}\sinh kt$	$\frac{k}{(s-a)^2-k^2}$	(24)
$e^{at}\cosh kt$	$\frac{s-a}{(s-a)^2-k^2}$	(25)
$t\sin kt$	$\frac{2ks}{(s^2+k^2)^2}$	(26)
$t\cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}$	(27)
$t\sinh kt$	$\frac{2ks}{(s^2-k^2)^2}$	(28)
$t\cosh kt$	$\frac{s^2-k^2}{(s^2-k^2)^2}$	(29)
$\frac{\sin at}{t}$	$\arctan\frac{a}{s}$	(30)
$\frac{1}{\sqrt{\pi t}}e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	(31)
$\frac{a}{2\sqrt{\pi t^3}}e^{-a^2/4t}$	$e^{-a\sqrt{s}}$	(32)
$\text{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$	(33)