National Exams December 2014

07-Elec-A2, Systems & Control

3 hours duration

NOTES:

1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, handwritten, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate answers to which questions should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

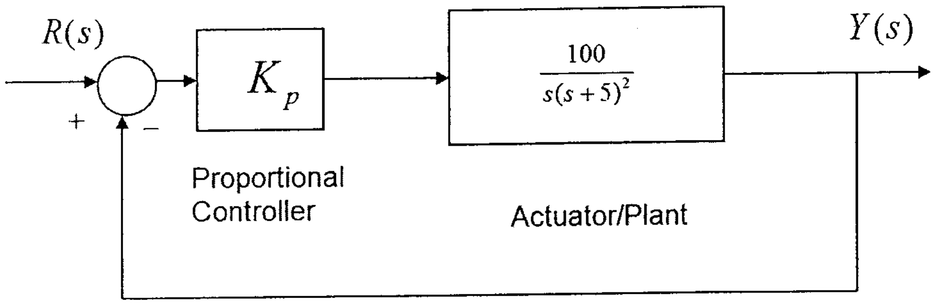
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{a}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{a}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t+\cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Question 1 (Compulsory)

Frequency Response, Routh-Hurwitz and Bode Criteria of Stability, Steady State Errors and Error Constants, System Type.

Consider a certain closed loop control system under Proportional Control, as shown in Figure Q1.1. Open loop frequency response plots of the system ($K_p = 1$) are shown in Figure Q1.2 and closed loop frequency response plots of the system ($K_p = 1$) are shown in Figure Q1.3.



Actuator/Plant transfer function is:

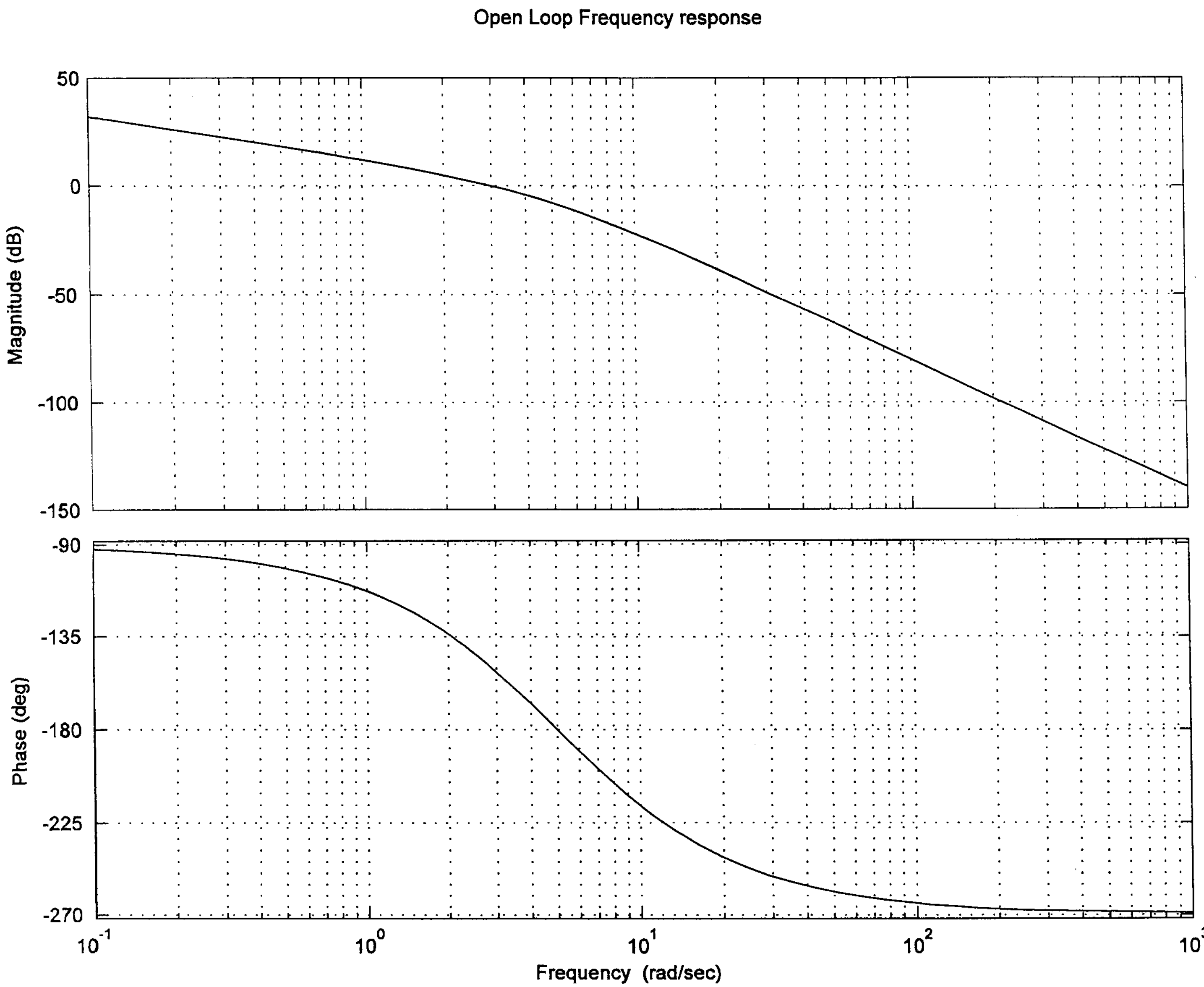
$$G(s) = \frac{100}{s(s+5)^2}$$

Figure Q1.1

1) **(5 marks)** Find the system Gain Margin, Phase Margin and corresponding crossover frequencies. Determine the critical value of the gain, K_{crit}, at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, ω_{osc}. Determine the range of gains K_p to provide a stable closed loop system response.

2) **(10 marks)** Verify the results above by applying the Routh-Hurwitz Criterion of Stability: find the critical value of the gain, K_{crit}, at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, ω_{osc}. How do they compare to item 1)?

3) **(5 marks)** For $K_p = 1$, determine the following closed loop steady-state response specifications: System Type, Error Constants and Errors.



$G_{m(dB)} =$

$G_{m(V/V)} =$

$\omega_{cg} =$

$\Phi_m =$

$\omega_{cp} =$

Figure Q1.2

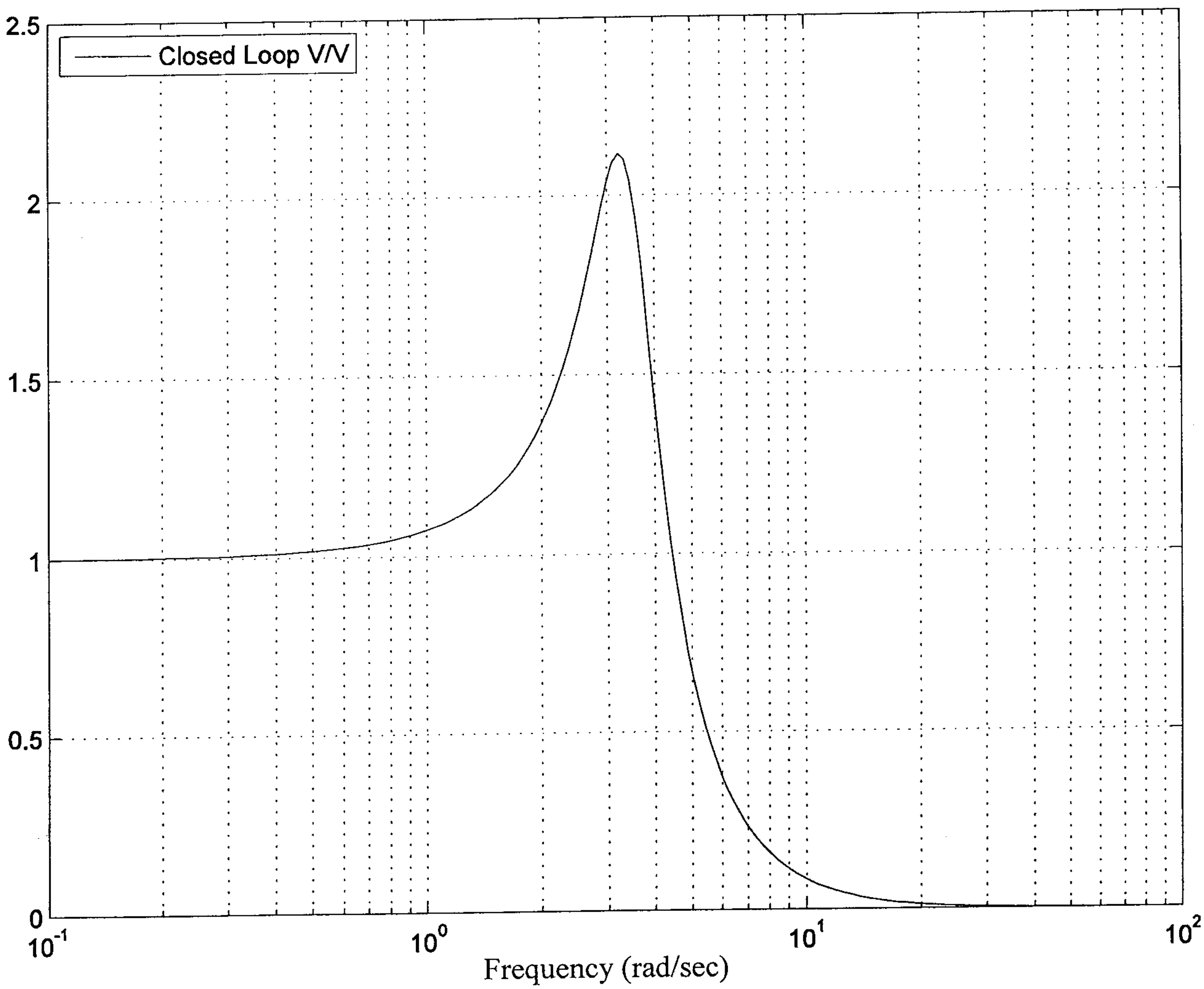


Figure Q1.3 - Closed Loop Frequency Response - Magnitude in V/V Units

Question 2 (Compulsory)

Root Locus Analysis and Gain Selection, PD Control, Response Specifications, Second Order Model.

Consider a certain closed loop unit feedback control system under Proportional + Derivative Control where the Derivative Time Constant, T_d, is equal to 2 seconds and where the Controller and the Process transfer functions are described as follows:

$$G_c(s) = K_p\,(1 + T_d s) \quad G(s) = \frac{2}{s^2(s+5)}$$

PART A (10 marks) - *Root Locus Analysis*

Sketch a Root Locus plot of the system - place it in Figure Q2.1. Find all relevant coordinates, including its centroid, asymptotic angles, if any, accurate coordinates of the break-away/break-in points, if any, accurate coordinates of crossovers with the Imaginary Axis, if any.

PART B (7 marks) - *Root Locus Gain Selection*

Use the Root Locus sketch to select a PD Controller Gain (K_p) such that the closed loop system would have the damping ratio of the dominant complex poles equal to $\zeta = 0.707$. Note that there are two possible choices of the Controller Gain that meet the ζ condition - clearly indicate their location on the Root Locus, then select the location that would result in a more desirable response and calculate the corresponding PD Controller Gain, K_p.

Briefly justify your choice of the Root Locus location for the Controller gain.

PART C (3 marks) - *Time Response Estimates*

Estimate the following compensated closed loop response specifications when the PD Controller Gain, K_p, has value as chosen in Part B: $PO, T_{settle(\pm 2\%)}, e_{ss(step\%)}, e_{ss(ramp)}, e_{ss(parabolic)}$.

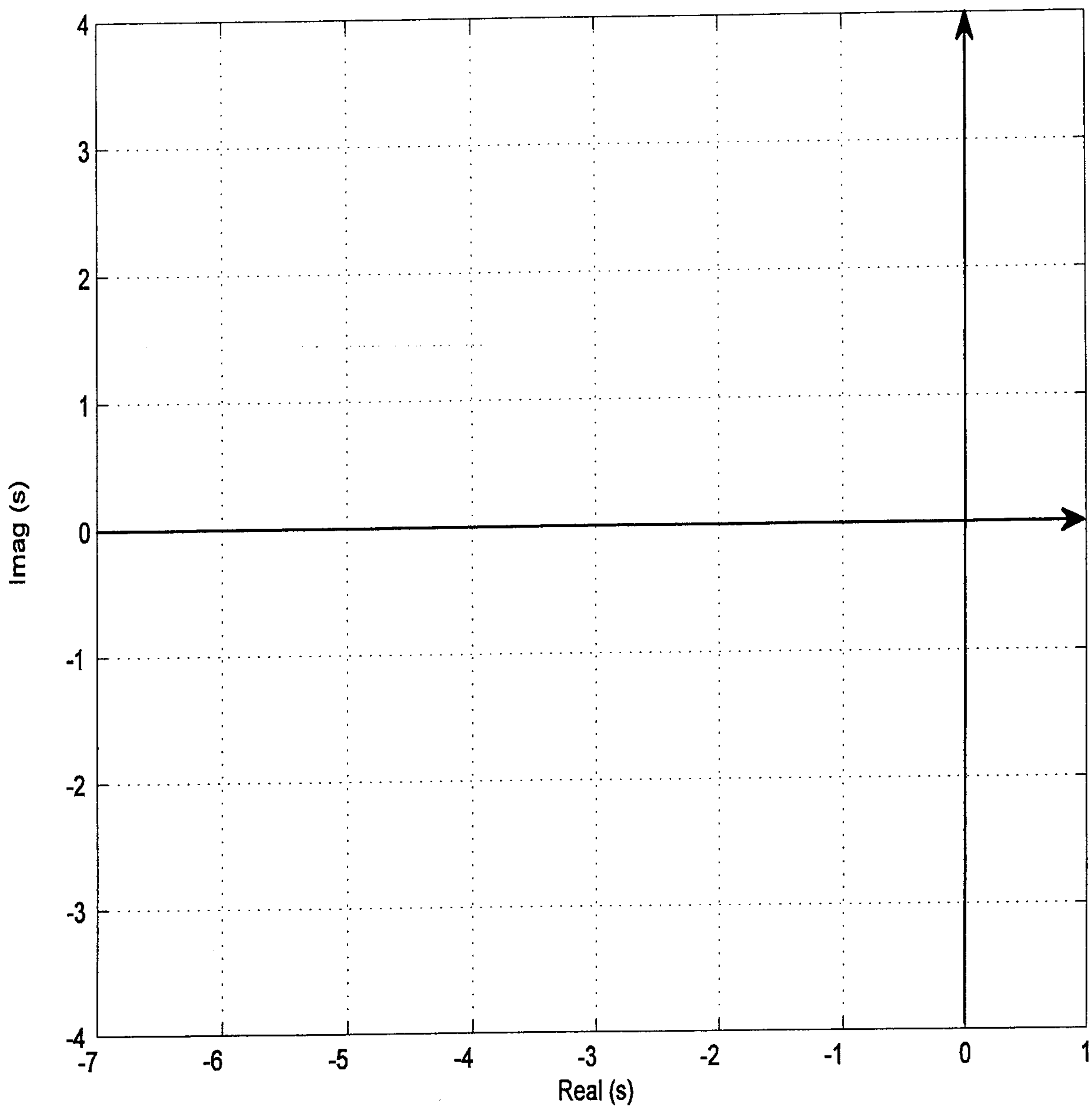


Figure Q2.1 - Place Your Root Locus Sketch Here

Question 3

Second Order Dominant Poles Models: from Pole-Zero Locations, from Open Loop Frequency Response and from Closed Loop Frequency Response, Step Response Specifications.

Consider the same closed loop control system under Proportional Control described in Question 1 and shown in Figure Q1.1. Recall that the open loop frequency response plots of the system ($K_p = 1$) are shown in Figure Q1.2 and the closed loop frequency response plots of the system ($K_p = 1$) are shown in Figure Q1.3. The closed loop transfer function of the system ($K_p = 1$) has been calculated and its three poles are factorized as follows:

$$p_1 = -0.7791 + j3.3524\,,\; p_2 = -0.7791 - j3.3524,\; p_3 = -8.4418.$$

1) **(5 marks)** Determine if a second order dominant poles model applies to the closed loop transfer function and if so, derive the model transfer function, $G_{m1}(s)$;

2) **(5 marks)** Assume the second order dominant poles model for the closed loop system, based on the information provided in the open loop Bode plots, and derive the model transfer function, $G_{m2}(s)$;

3) **(5 marks)** Assume the second order dominant poles model for the closed loop system, based on the information provided in the closed loop Bode plots, and derive the model transfer function, $G_{m3}(s)$;

4) **(5 marks)** How do the three models compare? Use the one you consider the most accurate to estimate the following closed loop step response specifications: $e_{ss(step\%)}$, $T_{rise(0-100\%)}$ and PO.

NOTE: The standard form of a second order model is as shown below, where ζ is a damping ratio, ω_n is a frequency of natural oscillations and K_{dc} is a DC gain of the model:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Question 4

PID Control Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system working under Proportional + Integral + Rate Feedback Control, as shown in Figure Q4.1:

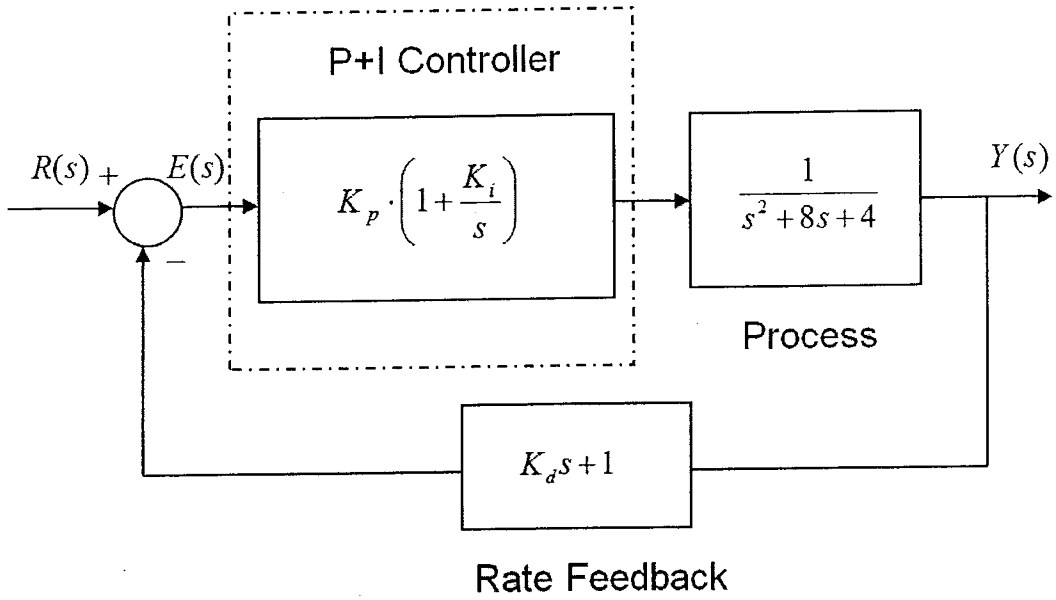


Figure Q4.1

1) **(5 marks)** Derive the closed loop system transfer function in terms of Controller Gains K_p and K_d and K_i.

2) **(5 marks)** It is desirable that the closed loop system settles within 1 second (assume the settling time definition within ±2% of the steady state) and that it has a Percent Overshoot of 10%. Determine the closed loop damping ratio, ζ, and the frequency of natural oscillations, ω_n, to meet the transient response requirements.

3) **(10 marks)** Choose the pole locations for the closed loop system so that the locations of its two complex conjugate poles correspond to the dominant poles based on the above model and the third real pole equals to the value of Integral Gain K_i so that a pole-zero cancellation in the closed loop transfer function occurs. Compute the required Controller Gains K_p and K_d and K_i.

Question 5

Transfer Function Calculations, Mason's Gain Formula, Solving for Step Response, Step Response Specifications.

Consider the block diagram of a servo-control system for one of the joints of a robot arm, shown in Figure Q5.1, where the input is the reference angular position for the robot arm, the output is the actual load angle of the arm, and the forward path contains a PD Controller, a calibration gain, motor and robotic arm dynamics and a gearbox. The Controller is described as:

$$G_c(s) = K_p + K_d s \qquad K_p = 15 \qquad K_d = 2.5$$

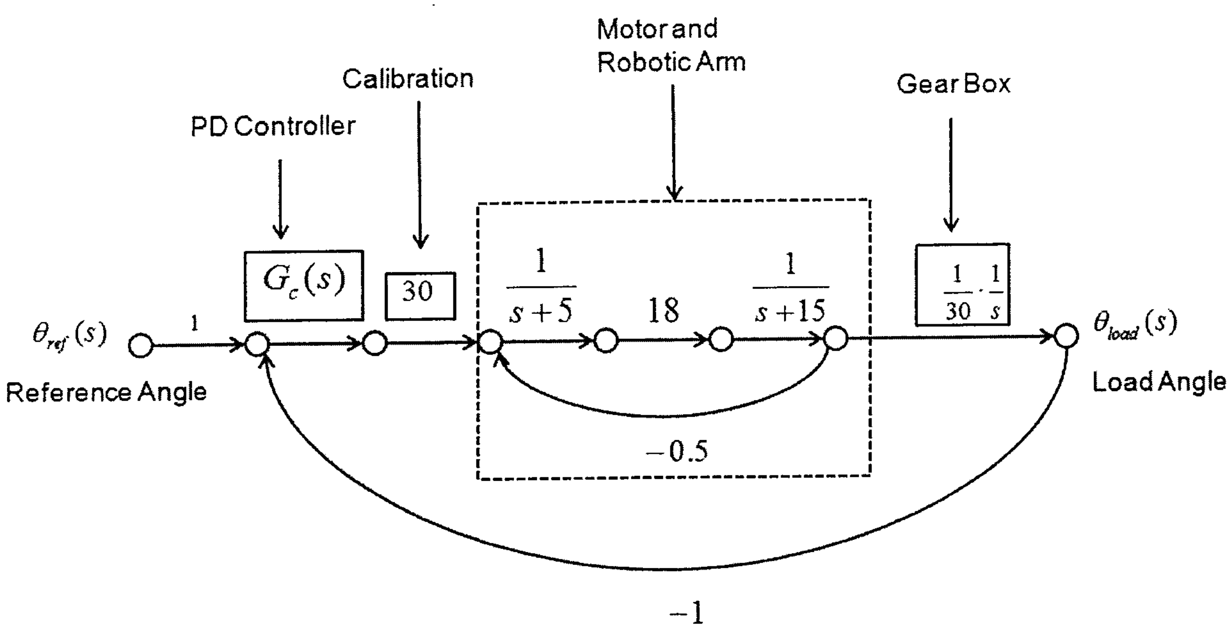


Figure Q5.1

1) **(10 marks) Find** the closed loop system transfer function, $G_{cl}(s)$

2) **(5 marks)** Determine the system Type and Error Constants, K_{pos}, K_v, K_a.

3) **(5 marks)** Find an expression for a unit step response of this system, $\theta(t), t \geq 0$.

HINT: one of the closed loop poles is at - 9.

Question 6

Controller Design in Frequency Domain, Response Specifications, Second Order Model.

You were in charge of a Controller design for a certain closed loop unit feedback control where the process transfer function was not known, but its open loop frequency response was successfully measured. Compensated system requirements were: Steady State Error to ramp input, $\mathbf{e_{ss(ramp)}} = \mathbf{0.1}$ V/V and Percent Overshoot, **PO** < 20%. Frequency responses of both the compensated and the uncompensated system are shown in Figure Q6.1.

Given that the system Type was identified as Type 1, you decided that a PI or a PID Controller is not necessary, and opted for the Controller configuration that could be considered as either a Lead or a Lag Controller, depending on the parameter values chosen:

$$G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}$$

PART A (8 marks) - *Time Response Estimates*

1. Use the plots in Figure Q6.1 to refresh your memory of the design, and remind us whether you used the Lead or the Lag Controller configuration by placing a check mark below:

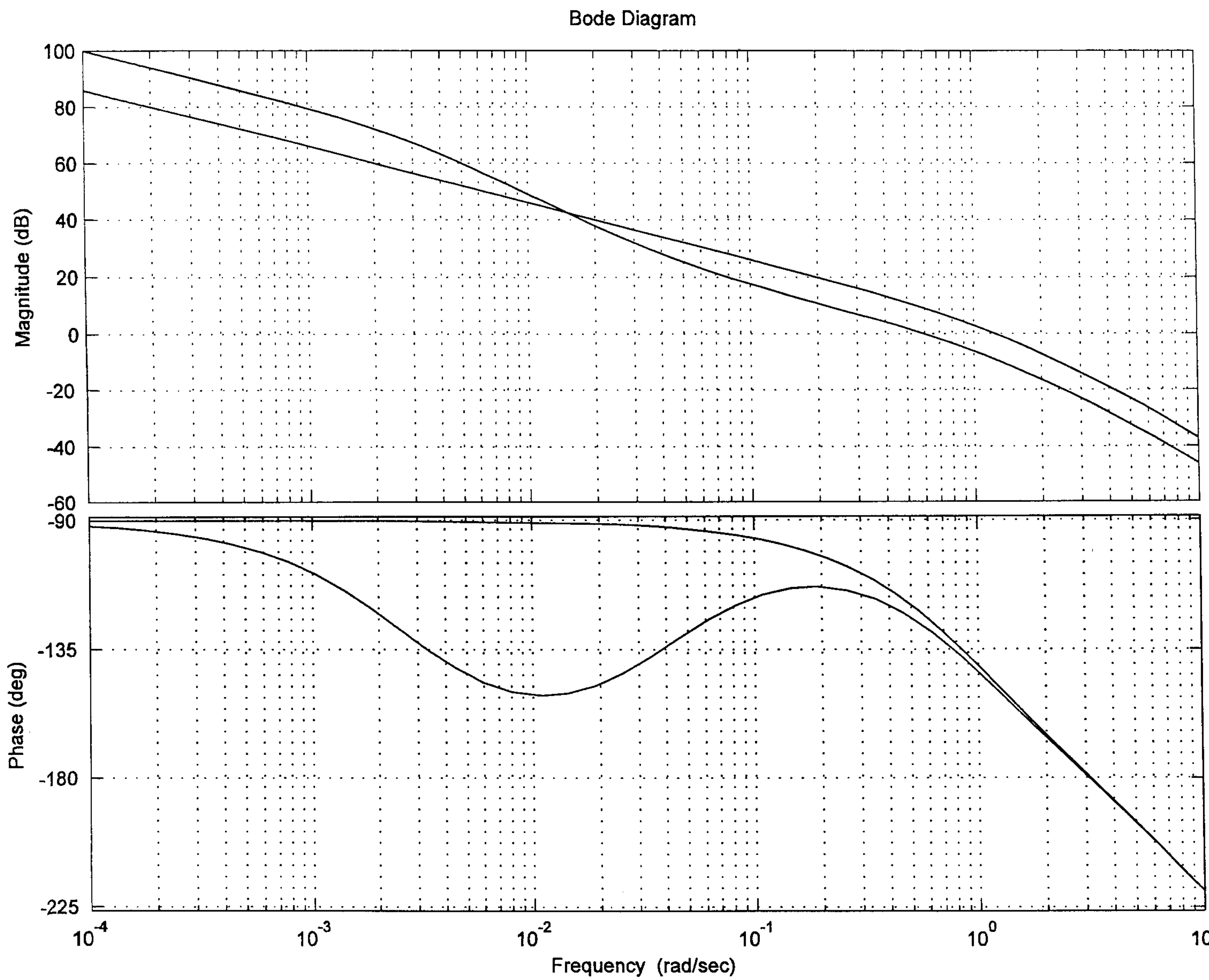
   ❑ LEAD CONTROLLER ❑ LAG CONTROLLER

2. Next, use the plots in Figure Q6.1 to read off the values of uncompensated Phase Margin (Φ_{mu}), uncompensated frequency of the crossover, (ω_{cpu}), uncompensated Velocity Error Constant (K_{vu}), and based on these, calculate these estimates for the **uncompensated** system: $e_{ss(ramp)}$, PO, and $T_{settle(\pm 2\%)}$.

3. Finally, use the plots in Figure Q6.1 to read off the values of compensated Phase Margin (Φ_{mu}), compensated frequency of the crossover, (ω_{cpu}), compensated Velocity Error Constant (K_{vu}), and based on these, calculate these estimates for the **compensated** system: $e_{ss(ramp)}$, PO, and $T_{settle(\pm 2\%)}$.

PART B (12 marks) - *Controller Design*

Show details of your chosen Controller design calculations, from the response specifications to the final Controller transfer function $G_c(s)$.



$\Phi_{mu} =$

$\omega_{cpu} =$

$K_{vu} =$

$\Phi_{mc} =$

$\omega_{cpc} =$

$K_{vc} =$

Figure Q6.1: Open Loop Frequency Response Plots of Uncompensated and Compensated Systems

Question 7

State Space Model, System Eigenvalues, Stability, Pole Placement by State Feedback Method, Steady State Errors to Step Inputs.

Consider a linear process described by the signal flow graph in Figure Q7.1:

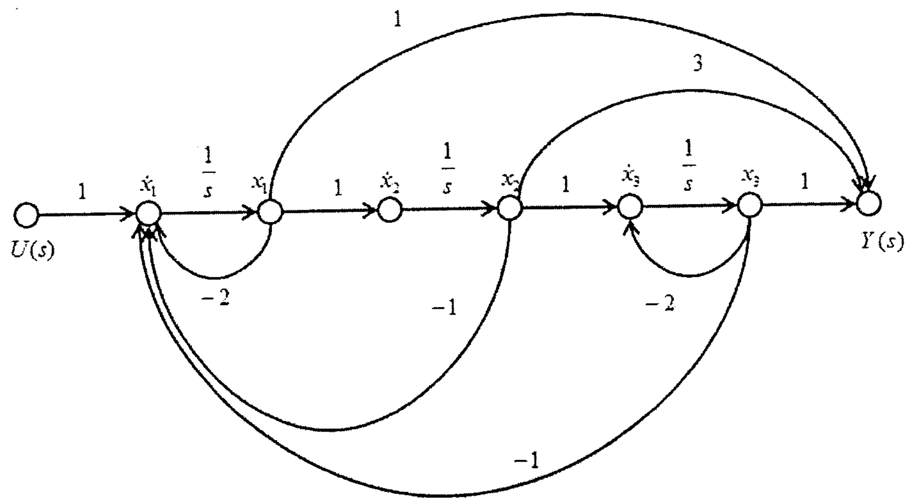


Figure Q7.1

PART A (10 marks)

Derive a set of the corresponding state equations - follow the choice of state variables as indicated in Figure Q7.1. Check if the process is Controllable and Observable.

PART B (10 marks)

A control system is to be built around the process by utilizing a state-variable feedback according to the following equation:

$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

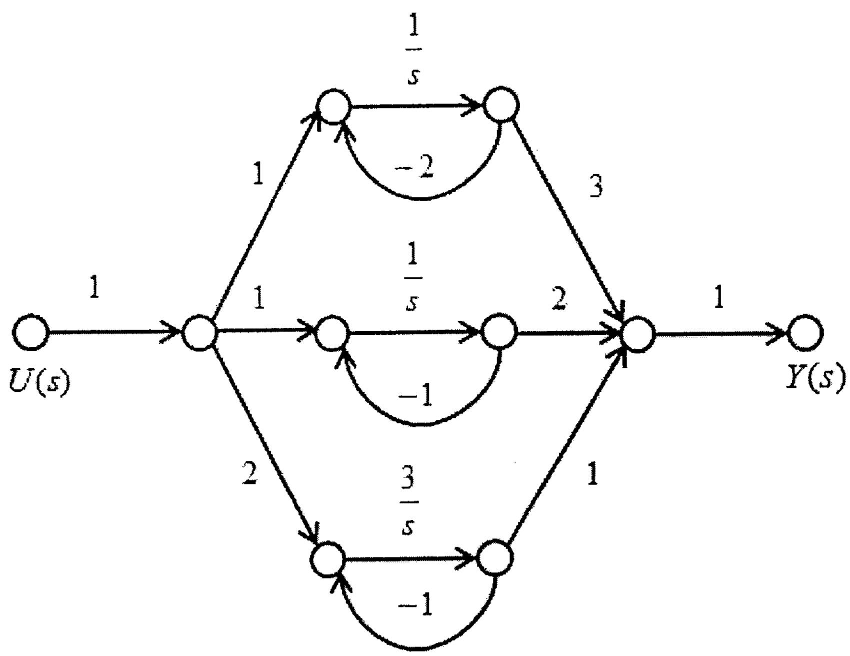
Determine the values of the gain constant K and the state feedback vector $\mathbf{k}$ so that the closed loop system will have poles at: -3 and at: $-2 \pm j3$, and the steady-state error to a step input signal r is to be zero.

Question 8

Mason's Gain Formula, Basic Analytical Relationships for a Standard Second Order System.

PART A (5 marks)

Consider the following signal flow graph, and derive the transfer function of a process G(s):



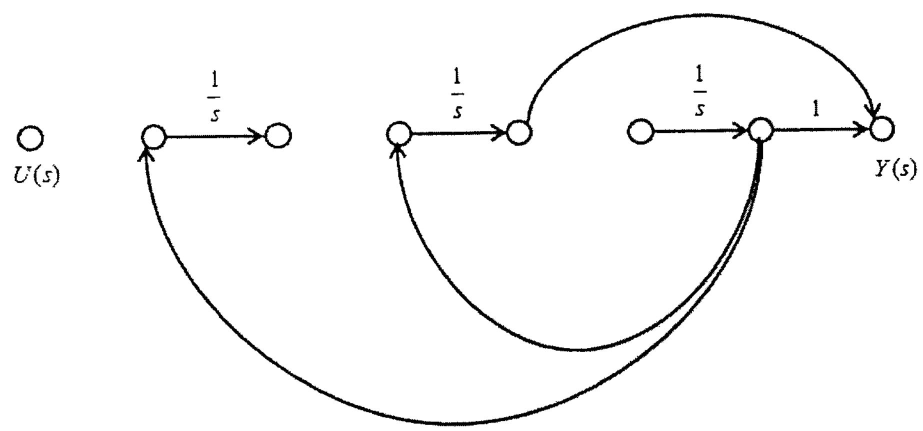
$$G(s) = \frac{Y(s)}{U(s)} = \rule{4cm}{0.4pt}$$

PART B (5 marks)

Consider the following transfer function of a certain process G(s):

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s + 20}{s^3 + 9s^2 + 26s + 24}$$

Complete the signal flow graph diagram shown so that it will represent G(s). Justify your sketch by applying the Mason's Gain formula to verify the transfer function.



PART C (10 marks)

Consider a second order under-damped system, described by the following transfer function:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ω_n represents the frequency of natural oscillations of the system, and ζ represents its damping ratio. Demonstrate how two well-known formulae describing relationships between system parameters and the system step response are derived:

$$PO = 100 \cdot \left(e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \right)$$

$$T_{settle(\pm 2\%)} = \frac{4}{\zeta\omega_n}$$

HINT: Use the Laplace Table entries to help with the derivations.